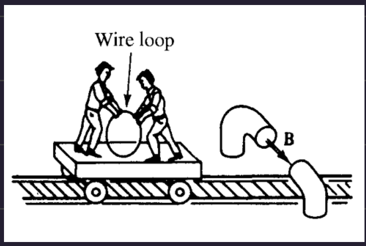
Einstein's Postulates

* <u>principle of relativity</u> : same laws apply in any inertial frame
  where Newton's 1st law holds.

→ 

motional emf established

$$\mathcal{E} = -\frac{d\Phi}{dt}$$ due to F_B on charges in the wire, moving along the train

same

* But in 'trolley' frame : loop at rest
  $\Rightarrow F_B = 0$

* but since $\vec{B}$ changes, $\Rightarrow \vec{E}$ induced
  $\Rightarrow F_E \neq 0 \Rightarrow \mathcal{E} = -\frac{d\Phi}{dt}$

⇒ Same result in both frames (though physical interpreta'ns of the process is completely wrong)

or is it?

* Both aren't wrong, their interpreta'ns differ.

→ Read about 'ether wind' and experimental testing, if interested.

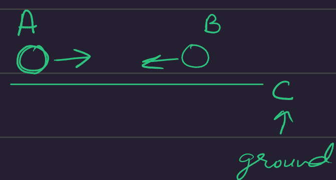
* Michelson- Morley experiments : to compare speed of light in diff. direc'ns (theoretically, different due to 'ether winds')

  * found : same in all directions!

→ <u>Einstein's Postulate</u> : i) <u>Principle of relativity</u> : laws of physics apply in all reference systems

ii) <u>Universal speed of light</u> : speed of light in vacuum is same for all inertial observers, regardless of the motion of the source.

* <u>Galileo's velocity addi'n rule</u> : $v_{AC} = v_{AB} + v_{BC}$



* <u>Einstein's " " "</u> : $$\boxed{v_{AC} = \frac{v_{AB} + v_{BC}}{1 + \frac{v_{AB}v_{BC}}{c^2}}}$$

(proved later)

Geometry of relativity

→ Relativity of simultaneity :

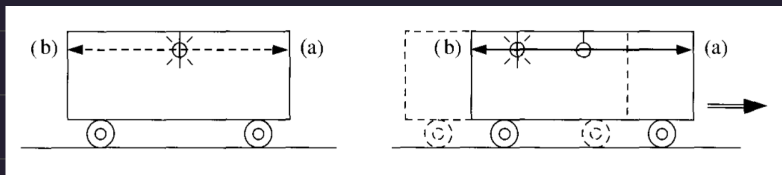


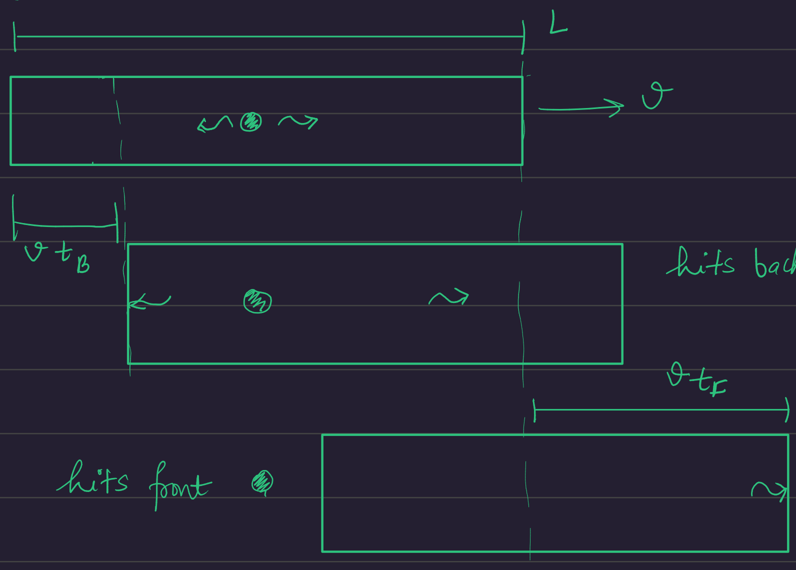
Figure 12.4

Figure 12.5

* 2 events → light reaches front
  ↳ light reaches end

* in frame of train, lamp is equidistant from 2 ends ⇒ events simultaneous !
* in platform frame, back end has shorter dist. than front end
  ⇒ not simultaneous !

* $\boxed{\text{In platform frame}}$: say t_F : when light reaches front
  t_B : ——— back



* $c t_B = \frac{L}{2} - v t_B$

$$\Rightarrow t_B = \frac{L/2}{c+v}$$

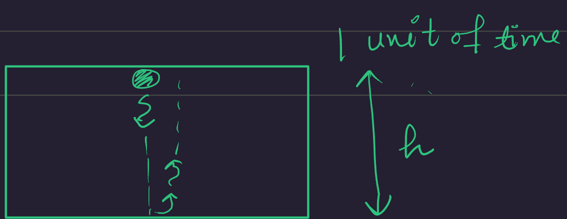
* $c t_F = \frac{L}{2} + v t_F$

$$\Rightarrow t_F = \frac{L/2}{c-v}$$

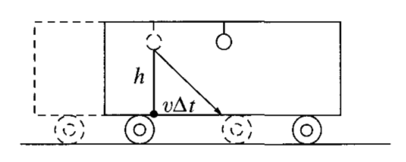
$$t_F - t_B = L/2 \left[\frac{1}{c-v} - \frac{1}{c+v} \right] = \frac{L}{2} \left(\frac{2v}{c^2-v^2} \right)$$

$$\Rightarrow \boxed{t_F - t_B = \frac{Lv}{v^2-c^2}}$$

→ Time Dilation : * Time clock :



$$\Delta \bar{t} = \frac{h}{c}$$



moving clock

$$\Delta t = \frac{\sqrt{h^2 + (v\Delta t)^2}}{c}$$

$$\Rightarrow \quad c^2(\Delta t)^2 = h^2 + v^2(\Delta t)^2 \Rightarrow \Delta t = \sqrt{\frac{h^2}{c^2 - v^2}}$$

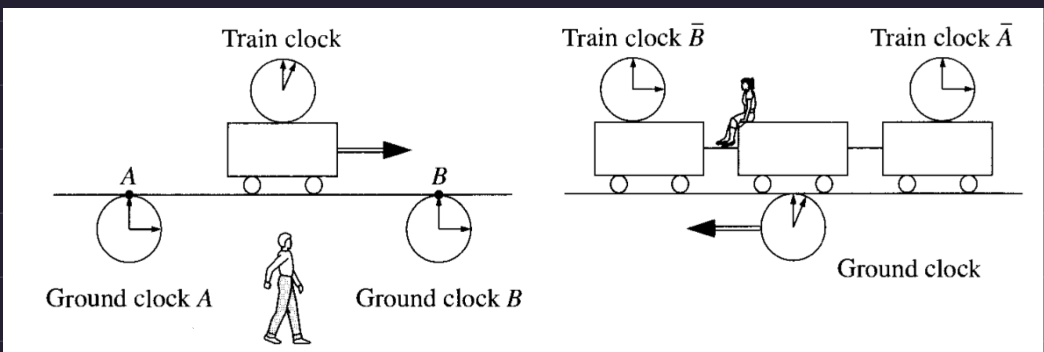
$$\Rightarrow \Delta t = \frac{h}{c}\,\frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\Rightarrow \boxed{\Delta t = \gamma\,\overline{\Delta t}\ ; \ \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}}$$

$\Rightarrow$ $\boxed{\text{Moving clocks run slow}}$ (time dilation)

* and from train's perspective, platform clocks are running slow

* How could they see each other's clocks slower than theirs? Isn't this contradiction to principle of relativity?
  * Arises due to relativity of simultaneity.
    * Ground observer is using clocks A & B, synchronized in its own frame to compare with the moving clock. Similarly, train observer uses C & D in its own frame.
    * But to ground observer, C & D aren't even synchronized because clocks synchronized in one frame aren't synchronized in other due to relativity of simultaneity.



→ * other forces of nature : fields on spacetime

* gravity : inherent in spacetime itself.

Newtonian

* $\vec{F} = \frac{GMm}{r^2} \vec{e}(r)$ & $\vec{F} = m\vec{a}$

OR $\nabla^2 \Phi = 4\pi G \rho$ & $\vec{a} = \vec{\nabla}\Phi$

grav. potential (Poisson's Eqⁿ)

Grav. field governed by matter

eqⁿ for response of matter to this field

To define GR, these to be replaced by statements about the curvature of space time.

* hard part :

In GR,

$$\underbrace{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}}_{\text{measure of curvature of spacetime}} = \underbrace{8\pi G\, T_{\mu\nu}}_{\text{measures energy \& momentum of matter}}$$

→ eqⁿ b/w 4×4 matrices

* response of matter to spacetime curvature : free particles move along paths of 'shortest possible distance' (geodesics)

* their parametrised paths $x^\mu(\lambda)$ obey the geodesic eqⁿ :

$$\frac{d^2 x^\mu}{d\lambda^2} + \Gamma^\mu_{\rho\sigma} \frac{dx^\rho}{d\lambda} \frac{dx^\sigma}{d\lambda} = 0$$

* metric tensor $g_{\mu\nu}$ underlies the descripⁿ of spacetime curvature, containing informaⁿ on how much the spacetime is altered from Euclidean by measuring deviaⁿ from Pythagoras thm ($(\Delta l)^2 = (\Delta x)^2 + (\Delta y)^2$)

Space – Time

→ * **Spacetime**: 4D set; elements: (t, x, y, z)

* **event**: point $\in$ spacetime

* **world line**: (path of particle) A parametrised 1D set of events

* in SR, no well-defined notion of two separated events occuring "at the same time" → depends on the coordinates used

* at any event, a **light cone**: locus of paths through spacetime that could concievably be taken by light rays passing through this event

$(v_{particle} < c \Rightarrow$ it moves along paths inside light cones$)$

*

for us in this discussion is the notion of an *inertial reference frame*, or IRF. Following Blandford and Thorne, we carefully define this frame as *a (conceptual) lattice of clocks and measuring rods that allows us to assign coordinates to (i.e., to label) spacetime events.* The IRF and this lattice have the following properties:

- The lattice moves freely through spacetime — no forces act on it, and it does not rotate relative to distant beacons.
- The measuring rods are orthogonal and uniformly ticked, forming an orthonormal, Cartesian coordinate system.
- All of the clocks tick uniformly.
- The clocks are synchronized using the "Einstein synchronization procedure": clock 1 emits a pulse of light at $t = t_e$. It bounces off a mirror on clock 2, and is received back at clock 1 at $t = t_r$. Clock 2 is synchronized with clock 1 such that the time of bounce is $t_b = (t_e + t_r)/2$. This synchronization is done between every pair of clocks in the lattice.

* **Clock Synchronization**:

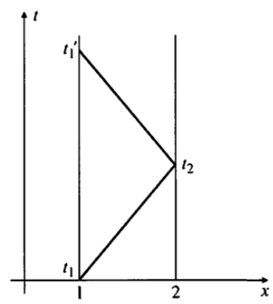


FIGURE 1.4 Synchronizing clocks in an inertial coordinate system. The clocks are synchronized if the time t_2 is halfway between t_1 and t_1' when we bounce a beam of light from point 1 to point 2 and back.

$$t_2 = \frac{t_1' + t_1}{2}$$

$\Rightarrow$ coord. system they constructed is an inertial frame

* this comparison is done locally and not with far-away clocks

* In SR, 3D space defined by $t = \text{const.}$ will differ from one defined when $t' = \text{const.}$

* spacetime interval :

defined as $(\Delta s)^2$ not (Δs)

$$\boxed{(\Delta s)^2 = -(c\Delta t)^2 + (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}$$

* $\exists$ a c s.t. spacetime interval is invariant under changes of inertial coordinates.

i.e. even in new inertial frame, (t', x', y', z')

$$(\Delta s)^2 = -c(\Delta t')^2 + (\Delta x')^2 + (\Delta y')^2 + (\Delta z')^2$$

$\Rightarrow$ makes sense to talk about SR as a theory of 4D space-time (Minkowski space)

* coordinates :

$$x^\mu : \quad \begin{aligned} x^0 &\equiv ct \\ x^1 &\equiv x \\ x^2 &\equiv y \\ x^3 &\equiv z \end{aligned}$$

* $c = 1$

* to refer to only space coord : x^i ; $x^1 = x$, $x^2 = y$, $x^3 = z$

* metric : (to write $(\Delta s)^2$ in compact form)

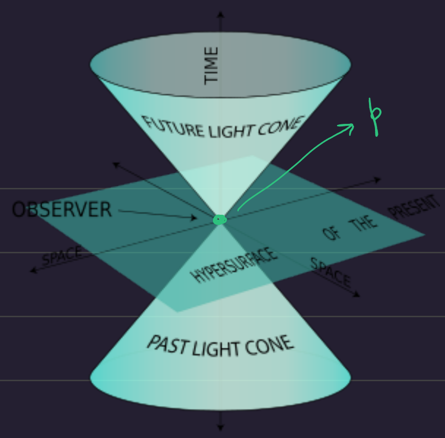
$$\eta_{\mu,\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow (\Delta s)^2 = \eta_{\mu,\nu} \Delta x^\mu \Delta x^\nu$$

dummy indices

Written in summation convention $\rightarrow$ indices appearing both as superscript and subscript are summed over.

* light cone :



set of all pts. connected connected to a single event by straight lines moving at the speed of light.

* time-like separated event from p :
$(\Delta s)^2 < 0$ inside time cone of p

* space-like " " " : $(\Delta s)^2 > 0$
outside " " " "

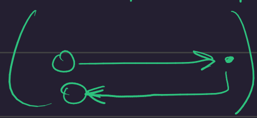
* light-like or null separated : $(\Delta s)^2 = 0$
on the time cone of p

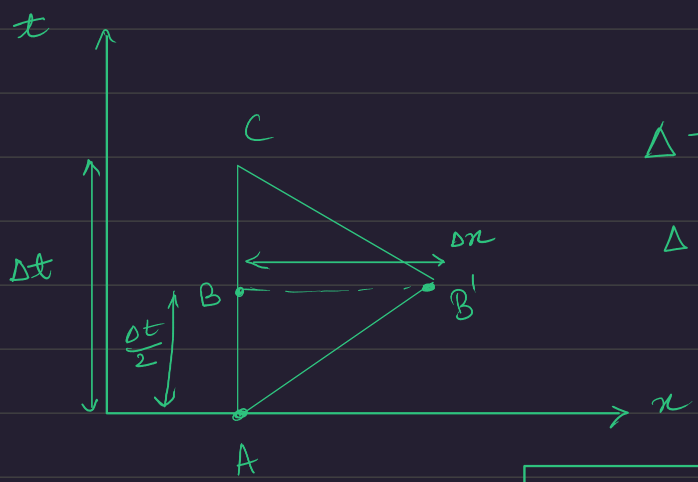
* proper time τ : $(\Delta \tau)^2 = -(\Delta s)^2$

* proper time b/w 2 events measures the time elapsed as seen by an observer moving on a straight path b/w the events

* $(\Delta s)^2$ invariant under changes of inertial frame

* eg 2 observers ⟶ stays at rest wrt. IFR
⟶ goes and comes back at the same spatial point





$$\Delta \tau_{ABC} = \Delta t$$

$$\Delta \tau_{AB'C} = 2\sqrt{\left(\tfrac{1}{2}\Delta t\right)^2 - (\Delta x)^2}$$

$$= \sqrt{1-v^2}\,\Delta t < \Delta t$$

$$\Rightarrow \boxed{\Delta \tau_{ABC} > \Delta \tau_{AB'C}}$$

Both aged different amounts!

* $\Delta \tau$ ⟶ time measured by an observer moving along the trajectory

* line element : $\boxed{ds^2 = \eta_{\mu\nu} \, dx^\mu dx^\nu}$

* Meaning of $\int \sqrt{\eta_{\mu\nu} dx^\mu dx^\nu}$ is unclear

instead, $x^\mu(\lambda)$ is taken (e.g. proper time for time like paths or an arbitrary parameter for general path)

This makes infinitesimal displacement along the curve more well-defined :

$$dx^\mu = \frac{dx^\mu}{d\lambda} d\lambda$$

$$\Rightarrow \quad \Delta s = \int \sqrt{\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}} \, d\lambda \quad \text{(for space like curve)}$$

$$\Delta \tau = \int \sqrt{-\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}} \, d\lambda \quad \text{(for time like curve)}$$

* massive particles move on timelike paths and massless move on null paths

Lorentz Transformation

* Way to relate various IRF, $\rightarrow (\Delta s)^2$ invariant
  * eg. translation

$$x^\mu \longrightarrow x^{\mu'} = \delta^{\mu'}_{\mu} (x^\mu + a^\mu)$$

$\hookrightarrow$ kronecker delta

* this keeps Δx^μ unchanged $\Rightarrow$ intervals unchanged

* eg. spatial rota's & offsets by a const. velocity vector (or boosts)
  * a linear transforma^n
  * $x^{\mu'} = \Lambda^{\mu'}{}_{\nu} \, x^\nu$ or conveniently, $x' = \Lambda x$ (matrix nota^n)

* here, Δx^{μ} not unchanged
* what matrix Λ will leave the interval variant?

(matrix notaⁿ) $$(\Delta s)^2 = (\Delta x)^T \eta (\Delta x) = (\Delta x')^T \eta (\Delta x')$$

$$= (\Delta x)^T \Lambda^T \eta \Lambda (\Delta x)$$

$$\Rightarrow \boxed{\eta = \Lambda^T \eta \Lambda}$$

or, $$\eta_{\rho\sigma} = \Lambda^{\mu'}{}_{\rho} \, \eta_{\mu'\nu'} \, \Lambda^{\nu'}{}_{\sigma}$$